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Estimates of the Nucleon Tensor Charge

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Like the axial vector charges, defined from the forward nucleon matrix element of the axial vector current on the light cone, the nucleon tensor charge, defined from the corresponding matrix element of the tensor current, is essential for characterizing the momentum and spin structure of the nucleon. These charges, which are first moments of the quark helicity and transversity distribution functions, can be measured, in principle, in hard scattering processes. Because there must be a helicity flip of the struck quark in order to probe the transverse spin polarization of the nucleon, the transversity distribution (and thus the tensor charge) decouples at leading twist in deep inelastic scattering, although no such suppression appears in Drell-Yan processes. This makes the tensor charge difficult to measure and its non-conservation makes it difficult to predict. While there may be constraints on the leading twist quark distributions through positivity bounds (e.g. the inequality of Soffer), there are no definitive theoretical predictions for the tensor charge, aside from model dependent calculations (e.g. the QCD sum rule approacch). We pursue a different route. Exploiting the approximate mass degeneracy of the light axial vector mesons ($a_1(1260)$, $b_1(1235)$ and $h_1(1170)$) and using pole dominance, we calculate the tensor charge. The result is simple in form and depends on the decay constants of the axial vector mesons and their couplings to the nucleons, along with the average transverse momentum of the quarks in the nucleon. The result is compared with other model estimates.

I. INTRODUCTION

The spin composition of the nucleon has been intensely studied. Its study has produced important and surprising insights, beginning with the revelation that the longitudinal spin is carried more by gluons than quarks. Considerable effort has gone into understanding, predicting

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and measuring the corresponding transversity distribution of the nucleon constituents. The leading twist transversity structure function, $h_1(x)$, is as fundamental to understanding the nature of the non-perturbative QCD regime of hadronic physics as is the longitudinal function $g_1(x)$, yet the transversity distribution can not be measured directly in deep inelastic lepton scattering since it is chiral odd.

Like the isoscalar and isovector axial vector charges defined from the forward nucleon matrix element of the axial vector current, the nucleon tensor charge is defined from the corresponding forward matrix element of the tensor current,

$$\langle P, S_T \left| \overline{\psi} \sigma^{\mu\nu} \gamma_5 \lambda^a \psi \right| P, S_T \rangle = 2\delta q^a \left(\mu_0^2\right) \left(P^\mu S_T^\nu - P^\nu S_T^\mu\right), \tag{1}$$

${S_T}^Q \sim (|+> \pm |->)$ for the moving nucleon is the *transversity*, a variable introduced originally by Moravcsik and Goldstein [1] to reveal an underlying simplicity in nucleon–nucleon spin dependent scattering amplitudes. Unlike the axial vector isovector and isosinglet charges, no sum rule has been written that enables a clear relation between the tensor charge and a low energy measurable quantity. So, aside from model calculations, there are no definitive theoretical predictions of the tensor charge. Among the various approaches, from the QCD sum rule [2–4], which estimates the flavor contributions to the tensor charge by analyzing the bilocal tensor current on the light cone, to lattice calculations [5] and light cone quark models [6], there appears to be a range of expectations and a disagreement concerning the sign of the down quark contribution.

Given the numerous experiments at, RHIC-Brookhaven (the PHENIX and STAR collaborations [7,8]), CERN (COMPASS experiment [9]), and HERA-DESY (the HERMES experiment [10]) and proposals [11,12] for extracting quark transversity distributions and, in turn, the flavor contributions to the nucleon's chiral odd tensor charge, there is reason to expect that in the not too distant future one will be able to reliably compare the data to the theoretical estimates of these quantities.

The various charges, which are first moments of the quark helicity and transversity distribution functions $\Delta q^a(x)$ and $\delta q^a(x)$ respectively, in principle can be measured in hard scattering processes. In their systematic study of the chiral odd distributions, Jaffe and Ji [13] related the

first moment of the transversity distribution to the flavor contributions to the nucleon tensor charge:

$$\int_0^1 \left(\delta q^a(x) - \delta\overline{q}^a(x)\right) dx = \delta q^a. \tag{2}$$

Because there must be a helicty flip of the struck quark in order to probe the transverse spin polarization of the nucleon, the transversity distribution (and thus the tensor charge) decouple at leading twist in deep inelastic scattering. No such supression appears in Drell-Yan scattering where Ralston and Soper [14] first encountered the transversity distribution entering the corresponding transverse spin (both the beam's and target's spin being transversly polarized to the incident beam direction) asymmetries .[1] Consequently, the charge is difficult to measure and its non-conservation [15] makes it difficult to predict. While bounds placed on the leading twist quark distributions through positivity constraints suggest that they satisfy the inequality of Soffer [16];

$$|2\delta q^a| \leq q^a + \Delta q^a, \tag{3}$$

(where q^a denotes the unpolarized quark distribution), model calculations yield a range of theoretical predictions [17].

Here we offer another estimate. Our motivation stems in part from the result that the tensor charge does not mix with gluons under QCD evolution and therefore behaves as a nonsinglet matrix element. [2] This, in conjuction with the fact that the tensor current is charge conjugation odd (it does not mix quark-antiquark excitations of the vacuum, since the latter is charge conjugation even, nor does it mix with gluonic operators under evolution, since gluonic operators are even), suggests that the tensor charge is more amenable to a valence quark model analysis. With this in mind we estimate the tensor charge by using axial vector dominance and an approximate phenomenological mass symmetry among the light axial vector mesons

[1] The quark lines are not correlated in the hard quark target amplitude.

[2] This is to be contrasted with the axial charges.

($a_1(1260), b_1(1235)$ and $h_1(1170)$). The b_1 and h_1 could couple to the quark tensor current in the nucleon at low energies, and via the symmetry, their coupling to the leptons is related to the a_1 production in τ decay.

Next we present our determination of the tensor charge under conditions of axial vector dominance in the context of our phenomenological symmetry represented by $SU(6)_W \otimes O(3)$ spin-flavor symmetry. We use this symmetry to relate the parameters of the previous section to measurable quantities. Then we present our results for the isoscalar and isovector and, in turn, up and down quark contributions to the the tensor charge of the nucleon. Finally we compare our results with QCD sum rule and other models calculations.

II. TENSOR CHARGE

We now apply axial vector dominance to estimate the tensor charge. That is, we choose to determine the value of the tensor charge at a scale where the matrix element of the tensor current is dominated by the lowest lying axial vector mesons [18]. Under these conditions, the matrix element of the tensor current, Eq. (1) becomes

$$\sum_{\mathcal{M}} \frac{\langle 0 \left| \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi \right| \mathcal{M} \rangle \langle \mathcal{M}, P, S_T | P, S_T \rangle}{M_{\mathcal{M}}^2 - k^2}. \tag{4}$$

The summation is over those mesons with quantum numbers, $J^{PC} = 1^{+-}$ that couple to the nucleon via the tensor current; namely the charge conjugation odd axial vector mesons – the isoscalar $h_1(1170)$ and the isovector $b_1(1235)$. To analyze this expression in the limit $k^2 \to 0$ we require the vertex functions for the nucleon coupling to the h_1 and b_1 meson and the corresponding matrix elements of the meson decay amplitudes which are related to the meson to vacuum matrix element via the quark tensor current. The former yield the nucleon coupling constants $g_{\mathcal{M}NN}$ defined from the matrix element

$$\langle MP | P \rangle = \frac{i g_{\mathcal{M}NN}}{2M_N} \overline{u}\left(P, S_T\right) \sigma^{\mu\nu} \gamma_5 u \left(P, S_T\right) \varepsilon_\mu P_\nu, \tag{5}$$

where P_μ is the nucleon momentum, and the latter yield the meson decay constant, $f_{\mathcal{M}}$

$$\langle 0 \left| \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi \right| \mathcal{M} \rangle = i f_{\mathcal{M}}^a \left(\epsilon_\mu k_\nu - \epsilon_\nu k_\mu \right), \tag{6}$$

where the k_μ and ε_ν are the meson momentum and polarization respectively. For transverse polarized Dirac particles, $S^\mu = (0, S_T)$ we project out the tensor charge using the constraint on the vector meson, $k \cdot \varepsilon_{\mathcal{M}} = 0$

$$\delta q^a(\mu^2) = \frac{f^a_{\mathcal{M}}\ g_{\mathcal{M}NN}\,(S_T \cdot k)\,\mathcal{A}}{P \cdot \varepsilon\ M^2_{\mathcal{M}}}, \tag{7}$$

where

$$\mathcal{A} = \frac{P \cdot \varepsilon\,(S_T \cdot k)}{2\ M_N} \tag{8}$$

is the nucleon-meson vertex function. In order to evaluate the tensor charge at the scale dictated by the axial vector meson dominance model we must determine the isoscalar and isovector meson coupling constants. We take a hint from the valence interpretation of the tensor charge and exploit the phenomenological mass symmetry among the lowest lying axial vector mesons that couple to the tensor charge; we adopt the spin-flavor symmetry characterized by an $SU(6)_W \otimes O(3)$ [19,20] mulitplet structure. Thus, the 1^{+-} h_1 and b_1 mesons fall into a $(35 \otimes L=1)$ muliplet that contains $J^{PC} = 1^{+-}, 0^{++}, 1^{++}, 2^{++}$ states. This analysis enables us to relate the a_1 meson decay constant measured in $\tau^- \to a_1^- + \nu_\tau$ decay [22,21]

$$f_{a_1} = (0.19 \pm 0.03)\mathrm{GeV}^2, \tag{9}$$

and the $a_1 NN$ coupling constant

$$g_{a_1 NN} = 9.3 \pm 1.0, \tag{10}$$

to the meson decay constants, f_{b_1} , f_{h_1} and coupling constants, $g_{b_1 NN}$ and $g_{h_1 NN}$. We find

$$f_{b_1} = \frac{\sqrt{2}}{m_{b_1}} f_{a_1}\,, \quad g_{b_1 NN} = \frac{5}{3\sqrt{2}} g_{a_1 NN}, \tag{11}$$

where the 5/3 appears from the $SU(6)$ factor (1+F/D) and the $\sqrt{2}$ arises from the L=1 relation between the 1^{++} and 1^{+-} states. [3] These, in turn, enable us to determine the isovector and isoscalar contributions

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[3]We note that our value of $f_{b_1} \approx 0.21 \pm 0.03$ agrees well with an independent sum rule determination of 0.18 ± 0.03 [23,4]

$$\delta q^v = f_{b_1} g_{b_1 NN} \frac{\langle k_\perp^2 \rangle}{\sqrt{2} M_N M_{b_1}^2}, \quad \text{and} \quad \delta q^s = f_{h_1} g_{h_1 NN} \frac{\langle k_\perp^2 \rangle}{\sqrt{2} M_N M_{h_1}^2} \tag{12}$$

respectively (where, $\delta q^v = (\delta u - \delta d)$, and $\delta q^s = (\delta u + \delta d)$). Transverse momentum appears in these expressions because the tensor couplings involve helicity flips that carry kinematic factors of momentum transfer. The intrinsic $k_\perp$ of the quarks in the nucleon is non-zero, as determined from Drell-Yan and heavy vector boson production processes. Using a Gaussian momentum distribution, and letting $\langle k_\perp^2 \rangle$ range from $\left(0.58\,\text{to}\,1.0\text{GeV}^2\right)$ [24] results in the up and down quark transversity ranging from

$$\delta u(\mu_0^2) = (0.53\,\text{to}\,0.92) \pm 0.20 \quad \delta d(\mu_0^2) = -(0.33\,\text{to}\,0.58) \pm 0.20. \tag{13}$$

This range of values for the u-quark tensor charge lies slightly lower than most other estimates while the d-quark charge is negative and of a comparable magnitude. Note that many predictions have the ratio $\delta d/\delta u$ near -1/4 or $(1-\sqrt{3})/(1+\sqrt{3})$, the value resulting from an $SU(3)$ degeneracy between the π^0 and the $\eta(8)$ octet elements in their coupling to the u-quark and the d-quark, i.e. the isoscalar coupling to u and d-quarks is $1\sqrt{3}$ times the isovector. In our calculation the isoscalar and isovector axial vector couplings to the nucleon also enter as factors in the expressions for the charges, with the D/F ratio being 3/2 in exact $SU(6)$. Loosening the $SU(6)$ constraint and incorporating mixing of the $h_1(1170)$ with the $h_1(1380)$ will alter the u to d ratio. These variations are being explored. Several other model calculations are summarized in the Table I.

δq^a	Lat	SR	BAG	CQ	QS	NJL
$\delta q_T^u(Q^2)$	0.84	1.33	1.09	1.19	1.07	0.82
$\delta q_T^d(Q^2)$	-0.23	0.04	-0.27	0.12	-0.38	-0.07

TABLE I. Sampling of estimates of the flavor contribution to the nucleon tensor charge calculated in: lattice (Lat) [5] ; QCD sum rules (SR) [2] ; the bag model (BAG) [25,2] ; the constituent quark model with Goldstone boson effects (CQ) [26] ; a quark soliton model calculation (QS) [27] ; and the NJL chiral soliton model (NJL) [28,29] with the associated momentum ranging from $0.40 - 1.0$ GeV^2 (errors range from 10-40%).

The calculation has been carried out at the scale $\mu_0 \approx 1$ GeV, which is set by the nucleon mass as well as being the mean mass of the axial vector meson multiplet. The appropriate evolution to higher scales (wherein the Drell-Yan processes are studied) is determined by the anomalous dimensions of the tensor charge [15] via

$$\delta q(\mu^2) = \left(\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right)^{\frac{4}{33-2n_f}} \delta q(\mu_0^2). \tag{14}$$

This is straightforward but slowly varying.

In conclusion, our axial vector dominance model with $SU(6)_W \otimes O(3)$ coupling relations provide simple formulae for the tensor charges. This simplicity belies the considerable subtlety of the (non-perturbative) hadronic physics that is summarized in those formulae. We obtain the same order of magnitude as most other calculation schemes. This result supports the view that the underlying hadronic physics, while quite difficult to formulate from first principles, is essentially a 1^{+-} meson exchange process. Forthcoming experiments will begin to test this notion.

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